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RADIATION SLIP IN A HIGHLY POROUS MATERIAL LAYER

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Expressions are obtained for the coefficient of radiant heat conductivity and the temperature jumps in the radiation slip mode in a highly porous material layer.

The model of a dust-laden gas [1] is often used as the globular model of a porous body in the theory of transport processes in highly porous media. Its crux is that the highly porous body is simulated by a homogeneous system of spherical particles of identical radius r distributed randomly and fixed in space (see sketch).

If the skeleton of such a porous body is opaque, while its material has the emissivity ε, then radiation transport therein can be described on the basis of an integral equation for the radiation energy emanating from unit volume of the porous body per unit time [3]. Assumptions about the homogeneity of the medium and isotropy of the scattering as well as the approximations of a gray body a and the photon mean free path Λ were used in deriving this equation. The radiation wavelength should here be much less than both the sphere diameter and the spacing between them.

The quantity Λ per unit volume equals [1]

$$\Lambda = \frac{4\Pi}{S} = \frac{4}{3}\frac{\Pi}{1-\Pi} r,$$

where $S = \dfrac{1-\Pi}{\frac{4}{3}\pi r^3} 4\pi r^2 = \dfrac{3(1-\Pi)}{r}$ is the surface of spheres of radius r.

If Λ is considerably less than the thickness of the porous layer L ($\ell = L/\Lambda \gg 1$) and the effective heat conductivity of the highly porous material is low and can be neglected then, the radiation energy transfer process can be considered diffusion. Using the approximation of an optically thick layer [4], the radiation transfer should be described by the heat conduction equation with radiant heat conductivity coefficient λ_R.

We write the expression for the energy flux density of the intrinsic radiation q_R in the section X in the form

$$q_R = 2\pi \int_0^X \int_0^1 S \frac{\sigma T^4(\xi)}{4\pi} \exp\left(-\frac{X-\xi}{\Lambda\mu}\right) d\mu d\xi -$$

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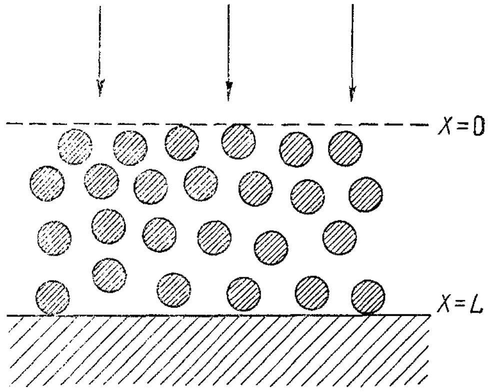

Fig. 1. Geometric diagram of the porous layer.

$$-2\pi\int\limits_X^L\int\limits_0^1 S\,\frac{\sigma T^4(\xi)}{4\pi}\exp\left(-\frac{\xi-X}{\Lambda\mu}\right)d\mu d\xi, \tag{1}$$

when deriving the relationship for λ_R in a highly porous body (by analogy with finding the diffusion coefficient in such a porous medium [5]) where $\mu = \cos\theta$ and θ is the angle between the x axis and the direction of photon motion.

After integrating (1) with respect to μ we obtain

$$q_R=\frac{1}{2}\int\limits_0^X S\sigma T^4(\xi)E_2\left(\frac{X-\xi}{\Lambda}\right)d\xi-\frac{1}{2}\int\limits_X^L S\sigma T^4(\xi)E_2\left(\frac{\xi-X}{\Lambda}\right)d\xi, \tag{2}$$

where $E_n(y)=\int\limits^1 \mu^{n-2}\exp(-y/\mu)\,d\mu$ is the exponential integral function. Assuming $E_2(y) = \frac{3}{4}\exp\left(-\frac{3}{2}y\right)$, expanding the function $T^4(\xi)$ in a Taylor series in the neighborhood of the point $\xi = X$ and limiting ourselves to the first two terms, we have for inner points of the layer ($X \gg \Lambda$) from (2)

$$q_R=-\frac{16}{3}\Pi\sigma T^3\Lambda\frac{dT}{dX}=-\frac{64}{9}\sigma T^3\frac{\Pi^2}{1-\Pi}r\frac{dT}{dX},$$

i.e.,

$$\lambda_R=\frac{64}{9}\sigma T^3\frac{\Pi^2}{1-\Pi}r. \tag{3}$$

Therefore, for the porous body model under consideration under the assumption of isotropy of the scattering, the radiant heat conductivity coefficient λ_R is proportional to $\Pi^2/(1-\Pi)$ and does not contain ε. Let us note that the form of the expression for λ_R depends on the porous body model and the radiation transfer process therein [6, 7]. In particular, it is shown in [6] for compact layers of spheres that if a two-flux approximation is used in the derivation of λ_R, then $\lambda_R = 8\sigma T^3/(a + 2b)$ (a, b are the absorption and back-scattering coefficients, respectively). In application to the model being utilized here (with isotropic scattering, we can formally set $a = \varepsilon/\Lambda$, $b = {}^1/_2(1-\varepsilon)/\Lambda$). The λ_R is also independent of ε.

As is known [4], in addition to the limit modes of optically thick and optically thin layers, the radiation slip mode associated with the deviation from the conditions of an optically thick layer at the boundaries of the medium, is also utilized in the theory of radiation transport. The temperature jumps on the interfacial boundaries under conditions of a non-heat-conducting medium are analogous in their nature to the temperature jumps in a rarefied gas [8]. In the simplest case their derivation is based on equilibrium of the sum of unilateral heat fluxes on the interfacial boundaries of the media to the resultant heat flux. Let us note that the concept of a unilateral flux is used when writing the condition on the boundary with a vacuum for the diffusion equation of neutron transport [9] and in the kinetic theory of gases. An expression is obtained in [10] for the temperature jump of a gas at the surface of a porous body of capillary configuration.

Let us write the temperature jump conditions for a porous layer bounded on one side (X = 0) by a vacuum and on the other (X = L) by a continuous material whose emissivity equals ε_1 (see Fig. 1). The expression for the unilateral radiant energy flux from the porous body to the surface X = L can be obtained from the first component in the right side of the relationship (2). Then, having performed the same calculations as in the derivation of λ_R, we find $\Pi\sigma T^4 = (^1/_2)\lambda_R(\partial T/\partial x)$. The absorbed and emitted energy flux at the boundary X = L equal, respectively, $\varepsilon_1(\Pi\sigma T^4 - (^1/_2)\lambda_R(\partial T/\partial x))$ and $\varepsilon_1\Pi\sigma T_1^4$, where T_1 is the surface temperature of the continuous material. Consequently, the temperature jump condition at the boundary X = L takes the form

$$\varepsilon_1\left(\Pi\sigma T^4 - \frac{1}{2}\lambda_R\frac{\partial T}{\partial X}\right) - \Pi\varepsilon_1\sigma T_1^4 = -\lambda_R\frac{\partial T}{\partial X},$$

or

$$-\left(\frac{1}{\varepsilon_1} - \frac{1}{2}\right)\lambda_R\frac{\partial T}{\partial X}\bigg|_{X=L} = \Pi\sigma\left(T^4 - T_1^4\right)\big|_{X=L}. \tag{4}$$

The temperature jump condition is obtained in analogous manner on the boundary with the vacuum

$$\frac{1}{2}\lambda_R\frac{\partial T}{\partial X}\bigg|_{X=0} = \Pi\sigma\left(T^4\big|_{X=0} - T_c^4\right), \tag{5}$$

where T_c is the temperature of the external medium. The derived boundary conditions (4) and (5) can be utilized in the formulation of boundary value problems of radiation-conductive heat transfer in highly porous materials.

NOTATION

T is the temperature, δ is the Stefan–Boltzmann constant; Π is the porosity; L is the porous layer thickness; Λ is the photon mean free path; ε, ε_1 are the emissivities of the porous and continuous materials and λ_R is the coefficient of radiant heat conductivity.

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